

1. a) Show that the extremal of the functional $F = y'^2 - y^2 + 2xy$, $y(0)=0$, $y(\frac{\pi}{2})=0$ is given by $y = x - \frac{\pi}{2} \sin x$ (12)
- b) Prove that the shortest distance between the two points in a plane is a straight line using Euler's equation. (8)
- OR
2. a) Find the minimum surface area stretched over a given closed space curve C enclosing a domain D in the xy plane. (20)
3. a) Solve the laplace equation $\nabla^2 u = 0$ with the boundary conditions $u(0,y)=y^2/2$, $u(4,y)=y^2$, $u(x,0)=0$, $u(x,4)=8+2x$, using Liebmman's process where $i,j=1,2,3,4$ (20)
- OR
4. a) Solve $\nabla^2 u = -4(x^2 + y^2)$ over the square mesh $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary and mesh length 1 unit correct to one decimal place. (20)
5. a) Find the numerically smallest eigen values and eigen vectors of the matrix
- $$A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix} \text{ using Inverse power method.} \quad (10)$$
- b) Using Modified Euler's method, find $y(0.1)$, $y(0.2)$ for $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. (10)
- OR
6. a) Using Jacobi's method find all the eigen values and eigen vectors of the matrix
- $$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \quad (10)$$
- b) Using Picard's method, solve $y' = x^2 + y^2$, $y(0)=0$ up to third iteration and hence find $y(0.1)$ & $y(0.2)$. (10)

7. a) Solve $y'' - y + x = 0$, $y(0)=0$, $y(1)=0$ using Raleigh Ritz method. (13)

b) Using Gauss Jordan method, Solve $10x+y+z=12$, $2x+10y+z=13$, $x+y+5z=7$. (7)

OR

8. a) Find the root of $x^3 + x^2 - 1 = 0$ correct to four decimal places that lies between 0 and 1 using Muller's method. (10)

b) Using Relaxation method, Solve $12x+y+z=31$, $2x+8y-z=24$, $3x+4y+10z=58$ (10)

9. a) Evaluate $\int_{-1}^1 e^{-x^2} \cos x dx$ using three point Gaussian Quadrature formula. (10)

b) Evaluate $\int_1^{1.4} \int_2^{2.4} \frac{dx dy}{xy}$ using Simpson's rule and check the result by direct Integration. (10)

Wishing you All the Best
